

Asymmetric Supernovae, Pulsars, Magnetars, and Gamma-Ray Bursts

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ABSTRACT

We outline the possible physical processes, associated timescales, and energetics that could lead to the production of pulsars, jets, asymmetric supernovae, and weak γ -ray bursts in routine circumstances and to a 10^{16} G magnetar and perhaps stronger γ -ray burst in more extreme circumstances in the collapse of the bare core of a massive star. The production of a LeBlanc-Wilson MHD jet could provide an asymmetric supernova and result in a weak γ -ray burst when the jet accelerates down the stellar density gradient of a hydrogen-poor photosphere. The matter-dominated jet would be formed promptly, but requires 5 to 10 s to reach the surface of the progenitor of a Type Ib/c supernova. During this time, the newly-born neutron star could contract, spin up, and wind up field lines or turn on an $\alpha - \Omega$ dynamo. In addition, the light cylinder will contract from a radius large compared to the Alfvén radius to a size comparable to that of the neutron star. This will disrupt the structure of any organized dipole field and promote the generation of ultrarelativistic MHD Waves (UMHDW) at high density and Large Amplitude Electromagnetic Waves (LAEMW) at low density. The generation of these waves would be delayed by the cooling time of the neutron star \simeq 5 to 10 seconds, but the propagation time is short so the UMHDW could arrive at the surface at about the same time as the matter jet. In the density gradient of the star and the matter jet, the intense flux of UMHDW and LAEMW could drive shocks, generate pions by proton-proton collision, or create electron/positron pairs depending on the circumstances. The UMHDW and LAEMW could influence the dynamics of the explosion and might also tend to flow out the rotation axis to produce a collimated γ -ray burst.

Subject headings: supernovae: general – gamma rays: bursts – pulsars:general – ISM: jets and outflows

1. Introduction

Recent evidence has given support for the idea that the core-collapse process is intrinsically strongly asymmetric. The spectra of Type II and Type Ib/c supernovae are significantly polarized indicating asymmetric envelopes (Méndez et al. 1988; Jeffrey 1991; Höflich 1991; Trammel et al. 1993; Wang et al. 1996; Tran et al. 1997; Leonard et al. 1999). The degree of polarization tends to vary inversely with the mass of the hydrogen envelope (Wang, Wheeler & Höflich 2000). Pulsars are observed with high velocities, up to 1000 km s^{-1} (Strom et al. 1995). Observations of SN 1987A showed that radioactive material rapidly mixed out to the hydrogen-rich layers (Lucy 1988; Sunyaev et al. 1987; Tueller et al. 1991). Cas A shows rapidly moving oxygen-rich matter outside the nominal boundary of the remnant (Fesen & Gunderson, 1996) and evidence for two oppositely directed jets of high-velocity material (Reed, Hester, & Winkler 1999). High velocity “bullets” of matter have been observed in the Vela supernova remnant (Taylor et al. 1993). Other evidence shows that soft gamma-ray repeaters arise in very strongly magnetized neutron stars, “magnetars,” with dipole fields in the range 10^{15} G (Kouveliotou et al. 1998; Thompson et al. 1999). Theoretical models have shown that matter-dominated jets can cause supernova explosions (Khokhlov et al. 1999) and that a strongly asymmetric explosion can account for the outward mixing of ^{56}Ni in SN 1987A (Nagataki 1999). In addition, much attention has recently been paid to the issue of collimation of γ -ray bursts and the associated affect on energetics and observable properties in the context of high-redshift events (Kulkarni et al. 1999; Rhoads, 1999; Sari, Piran & Halpern 1999; Lamb 1999; Chevalier & Li 1999a,b) and the proposed correspondence of SN 1998bw with a much lower energy event (Galama et al. 1998). In this paper, we explore the physics that might unite all these areas.

The discovery of optical afterglows of γ -ray bursts has raised the estimates of the maximum isotropic γ -ray emission to unprecedented values, $\sim 3 \times 10^{54} \text{ ergs}$ for GRB 990123 (Kulkarni, et al. 1999). This has brought a new focus on the likelihood that the prompt γ -ray bursts and the afterglows are collimated to various extents as well as Doppler boosted and “beamed” (Rhoades 1997, 1999; Sari, Piran & Halpern 1999; Kulkarni et al. 1999; Stanek et al. 1999; Harrison et al. 1999). Although the evidence is still preliminary, collimation factors of $\Delta\Omega/4\pi \lesssim 0.01$ have been derived from the decline in some afterglow light curves. Wang & Wheeler (1998), Nakamura (1998) and Cen (1998) have pointed out that if the collimation were strong enough, the energetics might be provided by supernova-like energies. SN 1998bw and GRB 980425 provided a different perspective by suggesting that some γ -ray bursts are directly associated with some supernovae (Galama et al. 1998; see also Bloom et al. 1999; Reichart 1999; Germany et al. 1999; Galama et al. 1999; Wheeler 1999). If this association is correct, the “isotropic” γ -ray energy of GRB 980425 is only $\simeq 10^{48} \text{ ergs}$. Some models of SN 1998bw invoked especially large kinetic energies, in excess of 10^{52} ergs in spherically-symmetric models, to account for the bright light curve and high velocities (Iwamoto et al. 1998; Woosley, Eastman & Schmidt 1998), and others took note of the measured polarization and chemical structure to suggest that strongly asymmetric models could account for the observations with more “normal” energies (Höflich, Wheeler & Wang 1999;

Danziger et al. 1999). It is not clear that either class of models can naturally produce a γ -ray burst of $\sim 10^{48}$ ergs. The former models have been referred to as “hypernova” models, although that term was originally introduced (Paczyński 1998) to mean the generic high energy events associated with the afterglows of the classical γ -ray bursts at large distance.

Many of the popular models for creating γ -ray bursts are based on binary neutron stars (Paczyński 1986) or accretion onto black holes (Paczyński 1991; Woosley 1993). The latter, in particular, are popular because there is no limit, in principle, to the mass of the black hole and hence, again in principle, the energy that can be extracted. All the models struggle with the mechanism for turning the large energies into γ -rays. There is no reason why the energy flux from a black hole source cannot be collimated, as witness the jets from active galactic nuclei and some binary black hole sources. This could reduce the energy requirements even in black hole models. Some models explicitly include this collimation (MacFadyen & Woosley 1999). We note that even the superluminal jets from AGN and blazars have a maximum bulk Lorentz factor of about 10, while the γ -ray bursts require $\Gamma \gtrsim 100$ (Krolik & Pier 1991; Baring & Harding 1997; Piran 1999, and references therein). This may suggest that a qualitatively different mechanism is needed to generate the cosmic γ -ray bursts despite the attractive possibilities for black hole models.

Here we will attempt to see how far a more conservative model can go, both to produce asymmetric supernovae and perhaps to generate γ -ray bursts, by considering the effects of a newly-born pulsar. This is not a new idea (Ostriker & Gunn 1971; Bisnovatyi-Kogan 1971; Bisnovatyi-Kogan et al. 1975), but it is worth reconsidering in the context of the polarization of core-collapse supernovae, the growing evidence for “magnetars,” the growing understanding of γ -ray bursts and their afterglows, the likely association of SN 1998bw with GRB 980425, and the interesting possibility that supernovae can be induced by energetic jets arising from their cores. All these processes demand or suggest strong asymmetries. Questions arise as to whether these issues are related, whether γ -ray bursts of observed properties could be produced in this context, whether there are more than one type of γ -ray burst mechanism, and whether supernovae may, in some circumstances, produce the γ -ray bursts seen at high redshift. It is very plausible that the formation of a neutron star will engender collimated flow, including Large Amplitude Electromagnetic Waves (LAEMW: Usov, 1992, 1994; Blackman & Yi 1998). Here we will show that it is plausible that the properties of SN 1998bw, including a weak γ -ray burst, may be generated by the acceleration of a collimated shock down a density gradient and possible that a γ -ray burst visible at cosmic distances could be produced under some circumstances. Certain aspects of our model are similar to those of Usov (1992, 1994), but he considered accretion-induced collapse and we explicitly investigate the context of a core collapse in the core of a massive star. Some of the ideas we explore here are also presented by Nakamura (1998).

The purpose of this paper is to outline the basic time scales, energetics, and relevant physical processes in order to define areas that need more quantitative work. Key length and time scales and a summary of the core collapse ambiance are given in §2 and §3, respectively. The physics of the proto-neutron star phase including the generation of the magnetic field and associated

torquing of the surrounding plasma is presented in §4. The effect of an axial jet associated with the collapse process is outlined in §5. The important phase when the neutron star cools, contracts, and spins more rapidly is discussed in §6, and the manner in which the energy associated with matter and radiation jets could be propagated outward is given in §7. Discussion and conclusions are given in §8.

2. Basic Length and Time Scales

In the discussion to follow there are a number of key length and time scales. Among these are the radius of the star, $R_{\text{star}} \simeq 10^8$ km for a red supergiant and $R_{\text{He}} \simeq 2 \times 10^5$ km for a helium core. The inner iron core that collapses to form a neutron star has a typical radius, $R_{\text{Fe}} \simeq 4 \times 10^3$ km. The dynamical or sound crossing time is

$$\tau_{\text{dyn}} \simeq 20s \frac{R_5^{3/2}}{(M/M_{\odot})^{1/2}}, \quad (1)$$

where R_5 is the radius in units of 10^5 km, and the light travel time is

$$\tau_{\text{light}} = \frac{R}{c} \simeq 0.3 \text{ s } R_5. \quad (2)$$

The dynamical time to emerge from a red giant or even a light crossing time, from 10^3 to 10^5 s, is not likely to be associated with observed γ -ray bursts. For a stripped helium core, the progenitor of a Type Ib or Type Ic supernova, a range of timescales from 1 to 10 s is possible, in the absence of Doppler boosting. Note that the breakout of a collimated jet might involve an area considerably less than the surface area of the helium core and an appropriately smaller time scale. Other relevant length scales are the radius of the light cylinder,

$$R_{LC} = \frac{c}{\Omega} = \frac{cP}{2\pi} \simeq 50 \text{ km} \left(\frac{P}{\text{ms}} \right), \quad (3)$$

where Ω is the rotational frequency and P the rotational period of the neutron star, and the Alfvén radius at which magnetic pressure is balanced by the ram pressure, e.g.,

$$\frac{1}{2}\rho v^2 \simeq \frac{1}{8\pi}B^2, \quad (4)$$

which, with $B \simeq B_{NS} \left(\frac{R}{R_{NS}} \right)^{-3}$, is

$$R_A \simeq 3.0R_{NS} \left(\frac{B}{10^{14} \text{ G}} \right)^{1/3} \left(\frac{\rho}{10^8 \text{ g cm}^{-3}} \right)^{-1/6} \left(\frac{v}{10^8 \text{ cm s}^{-1}} \right)^{-1/3}. \quad (5)$$

During the thermal contraction of the proto-neutron star, P , R_{LC} , and R_A will all change significantly. For illustration, we assume that the rotation period decreases from that of the

proto-neutron star, $P_{PNS} \simeq 25$ ms, to that of the neutron star, $P_{NS} \simeq 1$ ms. As a result, the light cylinder will contract from $R_{LC} \simeq 10^3$ km, beyond the radius of the stalled shock and comparable in size to the original iron core, to $R_{LC} \simeq 50$ km, comparable to the radius of the neutron star and well within the stalled shock. The light cylinder will contract from beyond R_A to significantly less than R_A . This has critical implications for angular momentum transport and the generation of radiation.

3. Context of Core Collapse

We know that pulsars arise from core-collapse. We know that some core collapse events are directly associated with supernovae, e.g., the Crab Nebula, SN 1987A. Circumstantial evidence suggests that supernovae of Type II and Ib/c result from core collapse based on correlations with spiral arms and H II regions in spiral galaxies and on the nebular spectra that are consistent with the cores of massive stars. Polarization data suggests that all these events are significantly asymmetric and that the asymmetry is stronger for explosions with smaller hydrogen envelopes (Wang, Wheeler & Höflich 1997; Wang & Wheeler 1998; Höflich, Wheeler & Wang 1999; Leonard et al. 1999; Wang, Wheeler & Höflich 2000). Explosions induced by jets from within the inner core can plausibly account for this asymmetry (Khokhlov et al. 1999). The imprint of such jets and the possibility to make γ -ray bursts is largest in massive stars from which the hydrogen envelope has been stripped by winds or binary mass transfer. In the following we will thus concentrate on core collapse explosions and neutron star formation in hydrogen and helium-deficient Type Ib and Type Ic supernovae. Such supernova progenitors must be surrounded by substantial mass from the previous mass loss stages, and this will affect the production and evolution of jets and γ -ray bursts.

Here we will adopt a generic picture of core collapse in an intermediate mass star of main sequence mass $\simeq 15 - 25 M_\odot$ with length scales as given in §2. The iron core undergoes instability and collapses in a time of $\simeq 1$ s. The collapse involves a homologous collapse of the iron core in which the density increases monotonically inward. The outer parts of the stellar core composed of silicon, oxygen, carbon, etc, have lower densities and longer free-fall times. Over the times of interest here, $\simeq 10$ s, these outer layers will “hover” as the collapsing iron core succeeds or fails to generate an explosion.

After core bounce, a proto-neutron star (PNS) forms with a radius of $R_{PNS} \simeq 50$ km and a shock is formed that stalls at a radius of $R_{sh} \simeq 200$ km. Over a cooling time $t_{cool} \simeq 5$ to 10 s, the proto-neutron star de-leptonizes by neutrino emission, cools, and contracts to form the final neutron star structure (Burrows & Lattimer 1986). If the neutron star is rotating and magnetized, this cooling phase will be associated with contraction, spin-up, and amplification of the magnetic field. These changes can significantly alter the physics associated with the neutron star and its interaction with its surroundings and hence the explosion process itself.

4. The Proto-Neutron Star Phase

Right after collapse, the hot proto-neutron star undergoes neutrino-driven convection with the characteristic convective overturn time scale $\tau_{\text{conv}} \simeq 1 \text{ ms } F_{39}^{-1/3}$ where $F_{39} \equiv F/10^{39} \text{ erg s}^{-1}$ is the neutrino flux at the base of the convection (Duncan & Thompson 1992). The hot proto-neutron star has a radius $\simeq 50 \text{ km}$ that gradually shrinks on the cooling time scale $\simeq 5$ to 10 s . This time scale is coincidentally, but significantly, similar to the sound crossing time of the helium core, and hence about the time required for a supersonic (but not relativistic) jet to penetrate the outer mantle. There are thus two potential mechanisms to create a γ -ray burst about 10 s after collapse. A γ -ray burst could be generated as the jet accelerates down the density gradient at the boundary of the core. Alternatively, a strong flow of Poynting flux might be generated on the neutron-star cooling time as the neutron star spins up. This might lead to an alternate mechanism of γ -ray burst formation on a similar timescale (Nakamura 1998). These mechanisms for generating γ -ray bursts will be discussed in §7.

It is hard to estimate the spin period of the proto-neutron star when it first forms. We assume that any γ -ray burst stimulated by Poynting from the neutron star will occur near the end of the cooling, contraction stage when the neutron star spins with a period $P_{NS} \simeq 1 \text{ ms}$. As we will see below, a spin period approaching $P_{NS} \simeq 1 \text{ ms}$ is necessary (if not sufficient) to power a classical γ -ray burst. A slower spin or shock breakout associated with a jet might generate the type of γ -ray burst observed in SN 1998bw/GRB 980425 as we will also show below. For purposes of illustration, we then adopt a spin period of the proto-neutron star to be that which will lead to a spin period of about 1 ms after cooling and contraction. Assuming angular momentum conservation during contraction of the original proto-neutron star, that $I_{PNS} \equiv kMR_{PNS}^2$ with $k = \text{const} \simeq \frac{2}{5}$, and that R_{NS} shrinks from 50 km to 10 km during the deleptonization phase, we take:

$$P_{PNS} \simeq 1 \text{ ms} \left(\frac{50 \text{ km}}{10 \text{ km}} \right)^2 \simeq 25 \text{ ms.} \quad (6)$$

We note that this spin rate for the proto-neutron star implies a rather rapid rate of spin for the original iron core. Conservation of angular momentum (assuming conservation of mass and that the form factor for the moment of inertia is unchanged) gives

$$\Omega_{Fe} \simeq \frac{M_{PNS}}{M_{Fe}} \left(\frac{R_{PNS}}{R_{Fe}} \right)^2 \Omega_{PNS} \simeq 0.04 \text{ s,} \quad (7)$$

or a period of about 2.6 minutes. This represents a rotational energy of about 10^{47} erg , an amount that is significant, but still much smaller than the binding energy of the iron core, $\simeq 10^{51} \text{ erg}$. If the total helium core is in solid body rotation with the same period of about 3 minutes, then for a mass of $5 M_{\odot}$ and a radius of $2 \times 10^{10} \text{ cm}$, the rotational energy would be about 10^{51} erg . This is clearly a demanding criterion. The specific parameters we adopt here in equation (6) and below are probably only reasonable if there is substantial differential rotation between the iron core and the surrounding star.

The moment of inertia of the proto-neutron star is $I_{PNS} \simeq 3 \times 10^{46}$ c.g.s. With the fiducial period from equation (6) the rotational energy of the proto-neutron star is then:

$$E_{\text{rot},PNS} \simeq \frac{1}{2} I_{PNS} \Omega_{PNS}^2 \simeq 9 \times 10^{50} \text{ erg} \left(\frac{M_{NS}}{1.5 M_{\odot}} \right) \left(\frac{\Omega_{PNS}}{250 \text{ s}^{-1}} \right)^2 \left(\frac{R_{PNS}}{50 \text{ km}} \right)^2. \quad (8)$$

This is a substantial energy, but since only a fraction of it could be tapped it is not clear that this energy could power a supernova never mind a classical γ -ray burst. If the rotational period of the proto-neutron star is longer, this is even more true. The fraction of this rotational energy that can be tapped depends on the behavior of the magnetic field, and we turn to that subject.

4.1. Magnetic Field Amplification

The magnetic field of the proto-neutron star is uncertain. If the pre-collapse core has a field strength comparable to that of a magnetized white dwarf, $\simeq 10^6$ G, then a field of $\simeq 10^{12}$ G could arise from flux-freezing. For $P_{PNS} \simeq 25$ ms (equation 6) and $\tau_{\text{conv}} \simeq 1$ ms, the Rossby Number $R_0 \simeq P_{PNS}/\tau_{\text{conv}} \simeq 25 \gg 1$ is too large to allow an $\alpha - \Omega$ type dynamo to operate. An alternative magnetic field amplification mechanism is linear amplification (Meier et al. 1976, Klužniak & Ruderman 1998). In this process, the differentially rotating neutron star could wrap the poloidal seed field into strong toroidal fields which then emerge from the star through buoyancy. After n_{ϕ} revolutions of the neutron star, the initial seed (poloidal) field is wrapped and amplified to produce a toroidal field

$$B_{\phi} \simeq 2\pi n_{\phi} B_p, \quad (9)$$

where B_p is the initial seed poloidal field. Buoyancy will operate to expel the field if the amplified final field B_f satisfies

$$\frac{B_f^2}{8\pi} \simeq f_B \rho c_s^2, \quad (10)$$

where $f_B \simeq 0.01$ is the fractional difference in density between the rising flux tube elements and the stellar material. Assuming sound speed $c_s \simeq c/3$ and density $\rho \simeq 10^{13}$ g cm $^{-3}$, one finds

$$B_f \simeq 2 \times 10^{16} \text{ G } f_{B-2}^{1/2} \rho_{13}^{1/2}, \quad (11)$$

where $f_{B-2} = f_B/0.01$ and $\rho_{13} = \rho / 10^{13}$ g cm $^{-3}$. Assuming that the magnetic flux tube (a torus) occupies a volume of $V_B/V_{PNS} \simeq 0.1$ where $V_{PNS} \simeq \frac{4\pi}{3} R_{PNS}^3$, the energy contained in the magnetic flux tubes at the buoyancy limit is estimated to be:

$$\begin{aligned} E_B &\simeq 0.1 \times V_{PNS} \times \frac{B_f^2}{4\pi} \\ &\simeq 1.6 \times 10^{51} \text{ erg.} \end{aligned} \quad (12)$$

This magnetic energy is ejected from the neutron star in the rising magnetic flux tubes. This energy is comparable to the proto-neutron star rotation energy (equation 8). For the

adopted parameters, the proto-neutron star would have to lose substantial rotational energy to the magnetic field before the field would float on dynamical time scales. This magnetic energy is still likely to escape from the proto-neutron star, but the details may be complex and involve the subsequent contraction of the neutron star.

The number of revolutions to reach $B_f \simeq 2 \times 10^{16}$ G is:

$$n_f = \frac{B_f}{B_0} \frac{1}{2\pi} \simeq 3 \times 10^3 \left(\frac{B_0}{10^{12} \text{ G}} \right)^{-1}. \quad (13)$$

For $B_0 \simeq 10^{12}$ G, the amplification time scale before the field is expelled by buoyancy is thus

$$t_f \simeq n_f P_{PNS} \simeq 25 \text{ ms} \times 3 \times 10^3 \sim 75 \text{ s}, \quad (14)$$

at the beginning of the contraction of the proto-neutron star. As the neutron star contracts, it spins up. The timescale for the linear field amplification, t_f , will decrease and the rotational energy of the neutron star will increase (cf §6). By the time the neutron star is spinning with a period of 1 ms, t_f would be substantially less than the cooling, contraction time of $\simeq 10$ s. Thus sometime during the contraction a dipole-type strong field is expected to be produced once the $\simeq 10^{16}$ G field emerges from the surface. A dipole field strength of $\simeq 10^{14}$ G is expected from the random sum of flux tubes of $\simeq 10^{16}$ G (Duncan & Thompson 1992; Thompson & Duncan 1993). It is likely that during the contraction phase a substantial fraction of $E_{rot,PNS} \simeq E_{B,PNS} \simeq 10^{51}$ erg would be extracted from the neutron star.

4.2. Energy Transport

We now consider the possible energy transport mechanisms from the neutron star to the outer stellar envelope. For $P \simeq 25$ ms, the radius of the light cylinder is (cf. equation 3) $R_{LC,PNS} \simeq 10^3$ km, comparable to the initial radius of the iron core and much larger than the proto-neutron star. For $R < R_{LC,PNS}$, a rotating dipole field could exert a magnetic torque on the plasma around the star. Typical collapse calculations (Burrows, Hayes & Fryxell 1996; Mezzacappa, et al. 1997) give for the density in the vicinity of the standing shock $\rho \simeq 10^8 \text{ g cm}^{-3}$ with a pre-shock velocity of $v \simeq 10^8 \text{ cm s}^{-1}$. Using those as fiducial values, the characteristic Alfvén radius for the proto-neutron star is (equation 5):

$$R_A \simeq 150 \text{ km} \left(\frac{R_{PNS}}{50 \text{ km}} \right) \left(\frac{B_{PNS}}{10^{14} \text{ G}} \right)^{1/3} \left(\frac{\rho}{10^8 \text{ g cm}^{-3}} \right)^{-1/6} \left(\frac{v}{10^8 \text{ cm s}^{-1}} \right)^{-1/3}. \quad (15)$$

Equation (15) implies that for B_{PNS} substantially less than 10^{14} G the Alfvén radius would be less than the radius of the proto-neutron star and hence that the magnetic torque would be ineffective. For $B_{PNS} \gtrsim 10^{14}$ G, the torque could have a significant effect. Equation (15) shows that $R_A \ll R_{LC} \simeq 1000$ km, so that a dipole field could be maintained over a significant volume. For the fiducial values chosen in equation (15), R_A is interestingly close to the stalled shock radius

$\simeq 200$ km, thus justifying the choice of density and velocity, to which R_A is not, in any case, very sensitive.

There are two relevant timescales pertinent to the action of the torque. One is the time to deflect infalling matter from radial infall to substantially azimuthal flow. The second is the time for this torque to spin down the proto-neutron star and hence to deposit energy in the infalling matter. The time for the torque to deflect infalling matter is approximately the momentum per unit volume of the infalling matter times an appropriate lever arm, R_A , divided by the torque per unit volume, $\simeq B_z(R)B_\phi(R) \simeq B^2(R)$ (cf. Shapiro & Teukolsky 1983). This effect of the torque thus operates on a time scale

$$\begin{aligned} t_{\text{tor},1} &\simeq \frac{\rho R_A v}{B^2} \simeq \frac{\rho R_A v}{4\pi \rho v^2} \simeq \frac{1}{4\pi} \frac{R_A}{v}, \\ &\simeq 0.01 \text{s} \left(\frac{R_{\text{PNS}}}{50 \text{ km}} \right) \left(\frac{B_{\text{PNS}}}{10^{14} \text{ G}} \right)^{1/3} \left(\frac{\rho}{10^8 \text{ g cm}^{-3}} \right)^{-1/6} \left(\frac{v}{10^8 \text{ cm s}^{-1}} \right)^{-4/3}. \end{aligned} \quad (16)$$

This time is sufficiently short that the torque could substantially alter the flow of matter, preventing the radial infall interior to the shock that is common to all spherically symmetric models of core collapse. Multi-dimensional models that produce non-radial circulation flows in the matter beneath the standing shock would also be substantially affected for conditions similar to those reflected in equation (16).

The time scale to extract energy from the proto-neutron star and deposit that energy in the surrounding plasma can be estimated by equating the torque, N , on the proto-neutron star to that on the gas,

$$N = I\dot{\Omega} \simeq \int_{R_A}^{\infty} R^2 B_z(R) B_\phi(R) dR \simeq R_A^3 B(R_A)^2. \quad (17)$$

Defining the timescale appropriate to this action of the torque to be $t_{\text{tor},2} = \Omega/\dot{\Omega}$ and using equation (17) gives:

$$\begin{aligned} t_{\text{tor},2} &\simeq \frac{1}{5\sqrt{\pi}} \frac{\Omega_{\text{PNS}} M_{\text{NS}}}{R_{\text{PNS}} B_{\text{PNS}} \rho^{1/2} v}, \\ &\simeq 170 \text{ s} \left(\frac{\Omega_{\text{PNS}}}{250 \text{ s}^{-1}} \right) \left(\frac{M_{\text{NS}}}{1.5 M_\odot} \right) \left(\frac{R_{\text{PNS}}}{50 \text{ km}} \right) \left(\frac{B_{\text{PNS}}}{10^{14} \text{ G}} \right)^{-1} \left(\frac{\rho}{10^8 \text{ gm cm}^{-3}} \right)^{-1/2} \left(\frac{v}{10^8 \text{ cm s}^{-1}} \right)^{-1}. \end{aligned} \quad (18)$$

The rate of energy deposition in the plasma by this torque is thus:

$$\begin{aligned} L_{\text{tor}} &= I\Omega\dot{\Omega} \simeq 2\sqrt{\pi}\Omega R_{\text{PNS}}^3 B_{\text{PNS}} \rho^{1/2} v, \\ &\simeq 1 \times 10^{49} \text{ erg s}^{-1} \left(\frac{\Omega_{\text{PNS}}}{250 \text{ s}^{-1}} \right) \left(\frac{R_{\text{PNS}}}{50 \text{ km}} \right)^3 \left(\frac{B_{\text{PNS}}}{10^{14} \text{ G}} \right) \left(\frac{\rho}{10^8 \text{ g cm}^{-3}} \right)^{1/2} \left(\frac{v}{10^8 \text{ cm s}^{-1}} \right). \end{aligned} \quad (19)$$

Note that since the torque is independent of Ω , the angular frequency will tend to decrease linearly. The timescale $t_{\text{tor},2}$ is sufficiently long and the associated energy deposition rate in equation (19) is sufficiently small that it is unlikely that this energy deposition will directly contribute to any supernova explosion. On the other hand, the selective deposition of this energy very near the stalled shock might have a leveraging effect belied by the small deposition rate that could help to reinvigorate a stalled shock in conjunction with other effects such as neutrino deposition.

5. The Effect of an Axial Jet

If, as in LeBlanc & Wilson (1970; see also Müller & Hillebrandt 1979; Symbalisty 1984), an MHD jet is formed during the collapse phase, the maximum jet power is estimated to be $E_{\text{rot}}/\tau_{\text{dyn}} \simeq 10^{51} \text{ erg}/1 \text{ s} \simeq 10^{51} \text{ erg s}^{-1}$. Although the details of the jet dynamics and energetics are not known, roughly $\lesssim 10^{51} \text{ erg s}^{-1}$ of power could be directed along the jet axis. The LeBlanc & Wilson calculation was criticized by Meier et al. (1976) as requiring extreme parameters of the progenitor star. These issues need to be re-examined in the current context, but we note several things about the Meier et al. analysis. They argue that the MHD axial flow found by LeBlanc & Wilson will not propagate to the stellar surface as a jet. The calculation of Khokhlov et al. (1999) shows that this is not necessarily correct, at least for a helium core. Meier et al. based their analysis on stellar evolution calculations of the day, but they adopted a stellar core with central density of about $10^{10} \text{ g cm}^{-3}$ giving a binding energy of about 10^{52} ergs . This exaggerates the binding energy of the initial core by about a factor of 10 compared to modern calculations and gives an incorrectly small value of a key parameter of Meier et al., the ratio of the binding energy of the newly-formed neutron star to that of the initial core. Meier et al. also did not consider the possibility of an $\alpha - \Omega$ dynamo that could lead to exponential field growth (Duncan & Thompson 1992). Symbalisty found that both substantial rotation and magnetic field were necessary to affect the explosion and that the presence of the magnetic field and associated losses allowed deeper collapse. The whole question of the initiation of MHD jets in association with neutron star formation needs to be considered anew. Here we consider some basic properties of jet propagation.

The jet would be stopped by the envelope when the jet is unable to provide the power to move the envelope material at a speed comparable to the jet velocity. This can be expressed as:

$$L_{\text{jet}} \simeq R^2 \Delta\Omega \times \rho_{\text{env}} v_{\text{jet}}^2 \times v_{\text{jet}}, \quad (20)$$

where $\Delta\Omega$ is the solid angle of the jet. The jet would then be stopped in a length

$$R_{\text{st}} \simeq \left[\frac{L_{\text{jet}}}{\Delta\Omega \rho_{\text{env}} v_{\text{jet}}^3} \right]^{1/2}. \quad (21)$$

In order to penetrate the star, the energy injected into the envelope at $R \lesssim R_{\text{st}}$ should be enough to unbind the region of the outer stellar mantle impacted by the jet. The amount of stellar material impacted by the jet is

$$\Delta M_{\text{env}} = M_{\text{env}} \frac{\Delta\Omega}{4\pi} \simeq 8 \times 10^{-3} M_{\text{env}} \left(\frac{\Delta\Omega}{0.1} \right). \quad (22)$$

Unbinding this mass requires

$$L_{\text{jet}} \gtrsim \frac{GM_{\text{env}} M_{\text{NS}} \Delta\Omega}{4\pi R_{\text{env}} \Delta t}, \quad (23)$$

where Δt is the duration of injection of the jet. Equation (23) can be expressed as:

$$L_{\text{jet}} \gtrsim 3 \times 10^{47} \text{ erg s}^{-1} \left(\frac{M_{\text{env}}}{1M_{\odot}} \right) \left(\frac{M_{\text{NS}}}{1.5M_{\odot}} \right) \left(\frac{R_{\text{env}}}{10^5 \text{ km}} \right)^{-1} \left(\frac{\Delta\Omega}{0.1} \right) \left(\frac{\Delta t}{1 \text{ s}} \right)^{-1}. \quad (24)$$

A jet of $\simeq 10^{51}$ erg s $^{-1}$ thus gives ample power to unbind a portion of the envelope occupying about 0.1 steradian.

The dynamics of the jet and impacted envelope material will depend on whether the mass of the jet is greater than or less than the mass of the displaced stellar envelope. The speed of the jet will be

$$v_{jet} \simeq 2c \left(\frac{2L_{jet}\Delta t}{M_{jet}c^2} \right)^{1/2}. \quad (25)$$

This implies a relativistic velocity for $L\Delta t \simeq 10^{51}$ ergs and $M_{jet} \lesssim 0.01M_{\odot}$. If a comparable amount of energy is put into the displaced envelope material, the impacted envelope mass, ΔM_{env} , would expand at a speed

$$v_{ej} \simeq \left(\frac{2L_{jet}\Delta t}{M_{env}} \frac{4\pi}{\Delta\Omega} \right)^{1/2}. \quad (26)$$

If this matter were displaced radially out of the star, it could also acquire relativistic speeds for $L\Delta t \simeq 10^{51}$ ergs and $\Delta\Omega \lesssim 0.1$. In practice, the displaced material will tend to be accelerated sideways by shocks induced by the passage of the jet and the energy will be shared by the whole envelope at roughly the sound speed in the envelope.

The propagation of the jet requires more study, but the calculation of Khokhlov et al. (1999) gives some qualitative insight into the behavior to be expected. As the jet propagates, a bow shock runs ahead of it. The speed of the bow shock is less than that of the matter inflow into the jet, a ratio of about 0.5 to 0.7 for the particular case explored by Khokhlov et al., but that ratio will depend on the speed and density of the jet, its opening angle, and the structure of the star through which the jet propagates.

The bow shock of the jet will both heat material and cause it to expand sideways. The opening half angle of the jet will then be approximately

$$\theta \simeq \frac{v_{env}}{v_{bow}} \simeq 0.1 \text{ rad} \frac{v_{env,8}}{v_{bow,9}} \simeq 5^o \frac{v_{env,8}}{v_{bow,9}}. \quad (27)$$

For $v_{env} \simeq 2 \times 10^8$ cm s $^{-1}$ and $v_{bow} \simeq 2 \times 10^9$ cm s $^{-1}$, $\Delta\theta \simeq 5^o$ or $\Delta\Omega/4\pi \simeq 0.004$. The actual dynamics of the jet will depend on nested cocoon-like shocks from the bow shock and subsequent expansion, as illustrated by Khokhlov et al. (1999).

6. The Neutron Star Phase

Concurrent with the propagation of any MHD jet and magnetic torque exerted by the proto-neutron star, the neutron star will contract and new physics can come into play. We assume that the cooling and contraction leads to a neutron star rotating at a period of about 1 millisecond. This implies a much smaller radius for the light cylinder,

$$R_{LC} = \frac{c}{\Omega_{NS}} \simeq 50 \text{ km} \left(\frac{P_{NS}}{1 \text{ ms}} \right). \quad (28)$$

This is only a little bigger than the neutron star radius. If the field remains about the same as the proto-neutron star, $B \simeq 10^{14}$ G, the Alfvén radius will be comparable to R_{LC} , but if the field amplifies during the contraction, as is plausible, the Alfvén radius will expand to become substantially greater than R_{LC} . Under this circumstance, a stationary dipole configuration at and beyond the Alfvén radius can no longer be maintained.

During the contraction phase, the Rossby number decreases in proportion to the rotational period. Assuming the neutron star convective time scale remains at about 1 ms, the condition $R_0 \lesssim 1$ will be reached if the neutron star ends up with a period of about 1 ms. This means that an $\alpha - \Omega$ -type dynamo could be initiated that would build a strong magnetic field up to $\sim 10^{17} - 10^{18}$ G (Duncan & Thompson 1992; Thompson & Duncan 1993; equipartition with the rotation would give $\sim 10^{18}$ G). In this extreme circumstance, the surface field could be $\sim 10^{15} - 10^{16}$ G for the dipole configuration near the neutron star surface (in the absence of the light cylinder). This phase of field growth occurs exponentially in contrast to the linear amplification associated with the relatively slowly-rotating initial proto-neutron star. If the dipole field grows to $B_{NS} \simeq 10^{16}$ G as the radius shrinks to $R_{NS} \simeq 10^6$ cm, then, all else being equal, the Alfvén radius will be $\simeq 140$ km, substantially larger than R_{LC} , as just noted. If the ram pressure has declined, R_A will be even larger.

The rotational energy after the contraction is

$$E_{rot,NS} \simeq \frac{1}{2} I_{NS} \Omega_{NS}^2 \simeq 6 \times 10^{52} \text{ erg} \left(\frac{M}{1.5 M_{\odot}} \right) \left(\frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right)^2 \left(\frac{R_{NS}}{10^6 \text{ cm}} \right)^2. \quad (29)$$

Setting aside the baryon-loading problem for the moment, this energy is comparable to the largest energy associated with any γ -ray burst, $\sim 3 \times 10^{54} \Delta\Omega/4\pi$ erg for GRB 990123 (Kulkarni, et al. 1999) for a degree of collimation, $\Delta\Omega/4\pi \lesssim 10^{-2}$. This degree of collimation has been deduced for some afterglows, in particular for GRB 990123 itself, and as noted in the previous section, is about the order expected for a matter-dominated jet (see also Khokhlov, et al. 1999). The degree of collimation of the subsequent flow of electromagnetic radiation is unclear. We return to that topic below.

The rotational energy of the contracted neutron star is radiated away in the form of intense waves of frequency Ω_{NS} generated at the speed of light circle. To generate traditional propagating electromagnetic waves as opposed to MHD waves, the displacement current must exceed the plasma current. This criterion can be written roughly as

$$\rho \lesssim \frac{B}{ecN_0P_{NS}} \simeq 10^{-6} \text{ g cm}^{-3} \left(\frac{B_{NS}}{10^{16} \text{ G}} \right) \left(\frac{P_{NS}}{1 \text{ ms}} \right)^{-1}, \quad (30)$$

where e is the electron charge and N_0 is Avogadro's number. The density within the star vastly exceeds this limit, so the energy will be generated as MHD waves. These waves will have extreme properties that require further study. In particular, the Alfvén speed can be estimated as

$$v_A = \left(\frac{B^2}{8\pi\rho} \right)^{1/2} \simeq 2 \times 10^{11} \text{ cm s}^{-1} \left(\frac{B_{NS}}{10^{16} \text{ G}} \right) \left(\frac{\rho}{10^8 \text{ g cm}^{-3}} \right)^{-1/2}. \quad (31)$$

If the surface field is of order 10^{16} G, then the field at the speed of light circle may be about 10^{15} G. Even for weaker fields, the density near the speed of light circle will decline in dynamic reaction to the large deposition of pulsar energy. Equation (31) thus serves to argue that this simple Newtonian expression for the speed of MHD waves is likely to be invalid since it is neither relativistic nor applicable in the strong field limit where the waves are not a first order perturbation to the plasma. Instead one must treat the generation and propagation of high amplitude, ultrarelativistic MHD waves (UMHDW), a task beyond the scope of this paper. Rather, we will sketch the basic energetics and the potential behavior of these waves.

If the density gets sufficiently low as the UMHDW propagate or because of dynamical reaction to energy deposition that creates a cavity causing the density to drop, then the plasma current can be exceeded by the displacement current and the waves can be described as Large Amplitude Electromagnetic Waves (LAEMW; Usov 1994; Blackman & Yi 1998). The properties of these waves are also not completely understood. Any LAEMW cannot propagate through a plasma if the plasma frequency

$$w_p = \left(\frac{4\pi n_e e^2}{m_e} \right)^{1/2} \simeq 9 \times 10^3 n_e^{1/2} \text{ Hz}, \quad (32)$$

exceeds the LAEMW frequency Ω_{NS} . The corresponding condition on the electron density is:

$$n_e \gtrsim 1 \text{ cm}^{-3} \left(\frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right)^2. \quad (33)$$

For densities exceeding this value, the LAEMW would have a very small skin depth and cannot propagate in the plasma. They would effectively be reflected by the plasma. The density exceeds this critical value under any stellar conditions.

In the discussion below we neglect the production of traditional Alfvén waves as an energy loss mechanism. Alfvén waves occur when the magnetic field is relatively weak and the waves are a propagating perturbation. Alfvén waves will thus be generated beyond the magnetosphere where the magnetic field no longer dominates the particle dynamics. In the proto-neutron star case, Alfvén waves could be generated in the region beyond the magnetosphere and within the extended light cylinder. The torque acting at R_A probably dominates those losses, but this is worth more detailed consideration. Once the neutron star contracts and the light cylinder passes within the magnetosphere, where Alfvén waves are not a relevant concept, the dominant losses will occur at the speed of light circle by generation of UMHDW. Alfvén waves might be generated up the rotation axis within the light cylinder but beyond the Alfvén radius. The global behavior of the open magnetic field lines that cross the light cylinder in the complex environment we are discussing is unclear. The dynamics of these open field lines again might lead to the production of Alfvén waves and associated losses, but it seems likely that in all these possibilities, Alfvén waves remain a secondary process.

The luminosity in UMHDW is estimated to be

$$L_{MHD} \simeq 4\pi R_{LC}^2 \times \frac{c}{4\pi} |\vec{E} \times \vec{B}| \simeq \frac{\mu_{NS}^2}{R_{LC}^4} c \simeq \frac{R_{NS}^6 B_{NS}^2 \Omega_{NS}^4}{c^3}, \quad (34)$$

assuming the UMHDW to be generated at R_{LC} and the magnetic moment of the neutron star to be $\mu_{NS} = B_{NS} R_{NS}^3$. For the conditions of the contracted neutron star which has initiated an $\alpha - \Omega$ dynamo, we expect

$$L_{MHD} \simeq 4 \times 10^{52} \text{ erg s}^{-1} \left(\frac{R_{NS}}{10 \text{ km}} \right)^6 \left(\frac{B_{NS}}{10^{16} \text{ G}} \right)^2 \left(\frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right)^4, \quad (35)$$

which will last for a duration of

$$t_{MHD} \simeq \frac{E_{rot,NS}}{L_{MHD}} \simeq 2 \text{ s} \left(\frac{M}{1.5 M_{\odot}} \right) \left(\frac{R_{NS}}{10 \text{ km}} \right)^{-4} \left(\frac{B_{NS}}{10^{16} \text{ G}} \right)^{-2} \left(\frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right)^{-2}. \quad (36)$$

The UMHDW are likely to initially be bottled up near the site of their production near R_{LC} since the surrounding plasma can respond only at the subrelativistic sound speed. The UHMDW would push the plasma aside and seek the easiest way out. An obvious possibility is that they will “burn” a channel along the rotation axis, following the path of the previous MHD jet. We assume that the most intense production of UMHDW occurs after the completion of the contraction of the neutron star, and hence that it is delayed by the cooling time $\simeq 5$ to 10 s, about the time necessary for the initial matter jet to propagate through the stellar core. A mass of about $\Delta M \simeq M_{env} \frac{\Delta \Omega}{4\pi} \simeq 10^{-2} M_{env}$ will be pushed aside by the initial matter jet. Whether the region impacted by the matter jet will be rarefied depends on the mass of the jet. If the jet is rather massive, as in the case of the calculation of Khokhlov et al. (1999), then the jet remains denser than the stellar environment as it propagates into the envelope. In this case, the jet acts as a “plug” until it disperses after several dynamical times. The UHMDW generated by the pulsar will tend to propagate through the lowest density regions. If there is a density minimum along the rotation axis, there will be a tendency for energy to flow in that direction. The dynamics are likely to be complicated and we will just sketch the possibilities.

The UMHDW should be rapidly isotropized in the co-moving frame, so they can be considered as a relativistic gas while they are trapped within the stellar core. It is not so clear that they are thermalized since their characteristic wavelength

$$\lambda_{MHD} \simeq c P_{NS} \simeq 2\pi R_{LC} \simeq 300 \text{ km}, \quad (37)$$

is comparable to or larger than the scale of the region in which they are generated. This is an important issue since if the energy is thermalized then the effective temperature will be,

$$T_{MHD} \simeq \left(\frac{E_{MHD}}{\frac{4\pi}{3} a R^3} \right)^{1/4} \simeq 2 \times 10^{10} \text{ K} E_{52}^{1/4} R_3^{-3/4}, \quad (38)$$

where R_3 is the radius in units of 10^3 km, so that much of the energy would go into the formation of pairs which might be dissipated by adiabatic expansion and difficult to recover. We will ignore this possibility for the moment, assuming that the energy cannot thermalize.

If the UHMDW act like a relativistic gas, then they will exert a pressure of

$$P_{MHD} \simeq \frac{1}{3} \left(\frac{E_{MHD}}{\frac{4\pi}{3} a R^3} \right) \simeq 8 \times 10^{26} \text{ erg cm}^{-3} E_{52} R_3^{-3}. \quad (39)$$

This may be contrasted to the pressure required to ensure hydrostatic equilibrium in the gravity of the neutron star which we can represent crudely as,

$$P_{HSE} \simeq \frac{GM_{NS}^2}{R^4} \simeq 1 \times 10^{28} \text{ erg cm}^{-3} \left(\frac{M_{NS}}{1.5M_{\odot}} \right)^2 R_3^{-4}. \quad (40)$$

By this measure, the pressure of the radiation will be less than required to produce HSE for small radii, and will exceed the pressure corresponding to HSE at a radius of

$$R_{HSE} \simeq 1 \times 10^4 \text{ km} \left(\frac{M_{NS}}{1.5M_{\odot}} \right)^2 E_{52}^{-1}. \quad (41)$$

The regime where the UMHDW begin to dominate will, of course, depend on the density profile in the collapsing matter and the whole environment is, in any case, dynamic so that considerations of HSE give only a partial perspective.

The production of the UMHDW near the light cylinder may lead to strong Rayleigh-Taylor instability as the relativistic fluid pushes on the surrounding gas. The subsequent behavior will depend on the rate at which the UMHDW “gas” expands quasi-spherically in Rayleigh-Taylor fingers compared to the rate at which the waves will selectively propagate upward along the rotation axis. It seems plausible that a substantial majority of the energy will flow up the axis, the path of least resistance. Another possibility is that the energy density becomes so high compared to the surrounding matter, near the neutron star or further out in the mantle (equation 41), that the stellar matter becomes an insubstantial barrier to the nearly free expansion of the UMHDW bubble. Since the total energy in the UMHDW could be as high as 10^{52} erg, this is a real possibility. Whether the UMHDW remain collimated may thus depend sensitively on the timescale on which they are generated and hence the instantaneous energy density throughout the star. These possibilities clearly need to be investigated with appropriate numerical calculations.

The result of the generation of the UMHDW could be rather different if they are thermalized. In this case, the UMHDW would be replaced with copious pairs (cf. equation 38). The pair gas would also act like a relativistic gas with the same issues of dynamics just discussed. In addition, there would be the complications of annihilation at boundaries with normal matter and the possibility of strong adiabatic losses after a pair jet broke out of the star and underwent free expansion. Such loss of pair thermal energy to rest mass and kinetic energy might be at least partially recovered if the pairs interacted with a surrounding wind of normal matter (see §8).

7. Evolution of Energy Emission and Gamma-Ray Bursts

There are two stages when γ -rays might be generated. The first is when the bow shock that proceeds the initial mass-dominated jet impacts on the stellar photosphere. The second phase is when a collimated flow of UMHDW erupts from the surface at about the same time. We consider those in turn.

7.1. Bow Shock Gamma-Ray Emission

Some γ -ray emission may be generated by the phase of shock breakout as the shock associated with the initial matter-dominated jet runs down the stellar density gradient and breaks through the photosphere. An associated mechanism is for the accelerated matter to reach relativistic speeds and then collide with some external matter.

As the bow shock that proceeds the jet runs down the exponential stellar density gradient in the photosphere it will accelerate, in turn heating and accelerating the matter, a mechanism to produce γ -rays first described by Colgate (1974, 1975; see Matzner & McKee 1999 for an updated discussion of analytic models of shock breakout). The question of how this matter expands and radiates and perhaps collides with an external environment is beyond the scope of this paper, but worth more detailed study. Here we remark on the basic properties at the time of shock passage.

Energy is deposited in the atmosphere by the bow shock. The bow shock, in turn, derives its energy from the jet. The power carried by the jet is $\simeq 1/2\rho_{jet}v_{jet}^3\Delta A$, where ΔA is the fractional area of the star that is impacted by the bow shock. This power will be delivered to the photosphere in a time $\simeq l_{phot}/v_{jet}$, where l_{phot} is the depth of the photosphere. The energy deposited in the photosphere during the bow shock break out phase is thus:

$$E \simeq \frac{1}{2}\rho_{jet}v_{jet}^2l_{phot}\Delta A \simeq 5 \times 10^{45} \text{ erg } \rho_{jet}v_{jet,10}^2l_{phot,7}\Delta A_{18}, \quad (42)$$

where $v_{jet,10}$ is the velocity of the bow shock in units of $10^{10} \text{ cm s}^{-1}$, $l_{phot,7}$ is in units of 10^7 cm and ΔA_{18} is in units of 10^{18} cm^2 . For the calculation of Khokhlov et al. (1999) the jet density is about $\rho_{jet} \simeq 10^3 \text{ g cm}^{-3}$, and the velocity steepens to about $90,000 \text{ km s}^{-1}$ as the jet approaches the outer density gradient of the helium core at which point the resolution of the density profile degrades. For $\rho_{jet} \simeq 10^3 \text{ g cm}^{-3}$, $v_{jet,10} \simeq 1$, $l_{phot,7} \simeq 1$, and $\Delta A_{18} \simeq 1$, the energy would be $\simeq 5 \times 10^{48} \text{ ergs}$, comparable to the γ -ray energy in the burst of SN 1998bw/GRB 980425.

The corresponding temperature, making the crude, and not necessarily correct, assumption of thermalization to a radiation-dominated gas, is

$$T \simeq \left(\frac{\rho_{jet}v_{jet}^2}{2a} \right)^{1/4} \simeq 10^8 \text{ K } \rho_{bow}^{1/4} v_{jet,10}^{1/2}, \quad (43)$$

For $\rho_{jet} \simeq 10^3 \text{ g cm}^{-3}$, $v_{jet,10} \simeq 1$, equation (43) gives $T \simeq 10^{10} \text{ K}$ with the possible emission of γ -rays.

This hot, accelerated matter would then expand and radiate. Whether it could account for the γ -ray burst in SN 1998bw/GRB 980425 will require more careful consideration. We note that if this energy accumulates in the density gradient of the photosphere the matter will be optically thin, so not susceptible to adiabatic losses while it undergoes free expansion. The photosphere is already optically thin by definition and the opacity will be decreased by Klein-Nishina corrections. On the other hand, the jet is denser and hence more opaque than the photosphere so it is important to know where the energy represented by equation (42) resides. Although the conditions could vary widely depending on the nature and propagation of the jet, for the calculation of Khokhlov et al. (1999), the bow shock material is matter ablated from the jet and the characteristic density is about the same as the jet, $\rho_{bow} \simeq 10^3$ g cm $^{-3}$.

The emission properties of the jet/bow shock/stellar atmosphere region as the jet impacts the atmosphere thus require more careful study. Nevertheless, considerable hard radiation could be emitted before the phase of homologous expansion is reached. The time scale for this energy to be radiated is also of importance. The shock break out time, $\simeq 10^{-3} l_{phot,7}/v_{jet,10}$ s, is far too short to correspond to an observed γ -ray burst, specifically that in GRB 980425, but the controlling time scale will be the radiative timescale.

Another possible means of producing γ -rays from the matter jet is to accelerate the matter in the photosphere to relativistic speeds and for it then to subsequently collide with a surrounding medium, perhaps a stellar wind. The mass fraction that can be accelerated in this way for an explosion with an impulsive energy input that drives a single shock through the outer stellar layers has been evaluated for CO and He stellar cores by Woosely, Eastman & Schmidt (1998). For their models, they establish a relationship

$$Q = \Gamma \beta (\rho r^3)^{0.2} \simeq 2.5 \times 10^6 \Gamma M_{ex,32}^{0.15}, \quad (44)$$

where the parameter Q is independent of Lagrangian mass and radius in the ejecta, $M_{ex,32}$ is the mass external to the layer with Lorentz factor Γ in units of 10^{32} gm, and we have taken $\beta \simeq 1$. Woosely et al. find that $Q \simeq 3 \times 10^5$. At the risk of overinterpreting their results, we note that for two otherwise identical models that differ only in the explosion energy input (models CO6A and CO6C), Q scales with the explosion energy, E_{ex} , approximately as $E_{ex}^{1/2}$. In the following we have adopted $Q \simeq 10^5 E_{ex,51}^{1/2}$, where $E_{ex,51}$ is in units of 10^{51} erg.

With this scaling, we find the kinetic energy in the homologously expanding material (e.g. after the immediate shock heating phase discussed above) to be

$$KE = \Gamma M_{ex} c^2 \simeq 1.6 \times 10^{44} \text{ erg } \Gamma^{-5.66} E_{ex,51}^{2.83}. \quad (45)$$

This expression is for a spherical explosion. It is clear that the energy in matter accelerated to relativistic speeds is insufficient to account even for the weak γ -ray burst imputed to GRB 990425, as concluded by Woosely, Eastman & Schmidt (1998).

In the present context, it is relevant to estimate the difference if the shock did not propagate spherically, but were collimated. In this case, we are interested in the kinetic energy in a jet with

solid angle $\Delta\Omega/4\pi$ or $KE_{\Delta\Omega} = KE\Delta\Omega/4\pi$. The relevant input energy is the energy in the jet, $E_{jet} = (\Delta\Omega/4\pi)E_{ex}$, where E_{ex} is now to be thought of as the equivalent isotropic energy of the collimated jet. Making this substitution in equation (45), we find:

$$KE_{\Delta\Omega} \simeq 7.2 \times 10^{47} \text{ erg } \Gamma^{-5.66} f_{coll-2}^{-1.83} E_{jet,51}^{2.83}, \quad (46)$$

where $f_{coll-2} = \Delta\Omega/4\pi$ in units of 10^{-2} . Equation (46) shows that even with substantial collimation, the amount of energy put into mildly relativistic matter ($\Gamma \gtrsim$ a few) by this mechanism is still insufficient to account for the putative γ -ray burst in SN 1998bw for an energy typical of the matter jet we have discussed here, $E_{jet,51} \simeq 1$. There might be sufficient energy for $E_{jet,51} \simeq 10$, but even this energy would be insufficient to boost enough stellar matter to $\Gamma \simeq 10$ to account for SN 1998bw.

From this analysis, we conclude that, even if it is strongly collimated, the physical process of running a single impulsively-induced shock (e.g. Colgate 1975; Matzner & McKee 1999) down the photospheric density gradient of a compact stellar core, accelerating that matter to relativistic speeds, and slamming it into a surrounding medium is unlikely to generate a substantial γ -ray burst of the kind associated with SN 1998bw. To produce a cosmic γ -ray burst with an energy of 10^{52} erg (isotropic equivalent of 10^{54} erg) and $\Gamma \gtrsim 100$ is out of the question. If jets from stellar collapse to produce either neutron stars or black holes are to generate γ -ray bursts in SN 1998bw, never mind the high redshift events, then the physics of the γ -ray production must be associated with the initial shock breakout phase (as discussed above) or by the prolonged emission phase of the jet passing through and beyond the photosphere.

7.2. The Contraction/UMHDW Phase

We now consider the time evolution of the contracting neutron star and the associated energy emission mechanisms implied by this scenario.

When $R_{LC} \gg R_A$, the magnetic torque is the dominant energy/momentum transport mechanism (§4.2). When $R_{LC} \ll R_A$, Poynting flux/UMHDW transports the energy/momentum. In our scenario, the proto-neutron star continues spinning up due to contraction even though it transfers angular momentum through magnetic torque. Initially, when $R_{LC} \gg R_A$, the rate of loss of energy to electromagnetic Poynting flux would be (cf. equation 35)

$$L_{EM} \simeq 2 \times 10^{46} \text{ erg s}^{-1} \left(\frac{R_{PNS}}{50 \text{ km}} \right)^6 \left(\frac{B_{PNS}}{10^{14} \text{ G}} \right)^2 \left(\frac{\Omega_{PNS}}{250 \text{ s}^{-1}} \right)^4. \quad (47)$$

This will be small compared to the energy deposited by the torque of the spinning neutron star acting at the Alfvén radius (equation 19),

$$L_{tor} \simeq 1 \times 10^{49} \text{ erg s}^{-1} \left(\frac{\Omega_{PNS}}{250 \text{ s}^{-1}} \right) \left(\frac{R_{PNS}}{50 \text{ km}} \right)^3 \left(\frac{B_{PNS}}{10^{14} \text{ G}} \right) \left(\frac{\rho}{10^8 \text{ g cm}^{-3}} \right)^{1/2} \left(\frac{v}{10^8 \text{ cm s}^{-1}} \right). \quad (48)$$

After contraction when the condition $R_{LC} \lesssim R_A$ is reached, we have:

$$\frac{R_A}{R_{NS}} \simeq 14 \left(\frac{B_{NS}}{10^{16} \text{ G}} \right)^{1/3} \left(\frac{\rho}{10^8 \text{ g cm}^{-3}} \right)^{-1/6} \left(\frac{v}{10^8 \text{ cm s}^{-1}} \right)^{1/3}, \quad (49)$$

and

$$R_{LC} \simeq 30 \text{ km} \left(\frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right)^{-1}, \quad (50)$$

and hence

$$\frac{R_{LC}}{R_A} \simeq 0.2 \left(\frac{R_{NS}}{10 \text{ km}} \right)^{-1} \left(\frac{B_{NS}}{10^{16} \text{ G}} \right)^{-1/3} \left(\frac{\rho}{10^8 \text{ g cm}^{-3}} \right)^{1/6} \left(\frac{v}{10^8 \text{ cm s}^{-1}} \right)^{1/3} \left(\frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right)^{-1}, \quad (51)$$

where ρ and v are to be evaluated at the Alfvén radius. Since energy injection during the phase when $R_{LC} > R_A$ will drive the density down in the vicinity of the magnetopause and the standing shock, and B_{NS} is likely to increase from 10^{14} G to 10^{16} G continuously (but suddenly) during the contraction by the linear amplification mechanism and dynamo, the epoch during contraction when $R_{LC} \simeq R_A$ will occur is difficult to estimate; however, since essentially all the energy emitted will be used to expel the envelope (i.e. either by torque or UMHDW) a more significant explosion/expansion is expected when $R_{LC} \lesssim R_A$. At this epoch we expect that if the torque were still active the rate of loss of rotational energy due to this mechanism would be

$$L_{tor} \simeq 3 \times 10^{49} \text{ erg s}^{-1} \left(\frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right) \left(\frac{R_{NS}}{10 \text{ km}} \right)^3 \left(\frac{B_{NS}}{10^{16} \text{ G}} \right) \left(\frac{\rho}{10^8 \text{ g cm}^{-3}} \right)^{1/2} \left(\frac{v}{10^8 \text{ cm s}^{-1}} \right). \quad (52)$$

This power would in any case be overwhelmed by the loss of energy to the UMHDW with the enhanced rotation and magnetic field,

$$L_{MHD} \simeq 4 \times 10^{52} \text{ erg s}^{-1} \left(\frac{R_{NS}}{10 \text{ km}} \right)^6 \left(\frac{B_{NS}}{10^{16} \text{ G}} \right)^2 \left(\frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right)^4. \quad (53)$$

This power output will last for a few seconds (equation 36). The injected energy is then $\gtrsim 10^{52}$ erg, which is enough to drive a significant shock. In this scenario, both a γ -ray burst and an asymmetric supernova could be produced during the “magnetar” phase.

The UMHDW will likely flow up the rotation axis starting with a cross-sectional area of $\simeq R_{LC}^2$ and rising as an intense photon “bubble.” Assuming that this bubble pushes matter aside at roughly the local sound speed, the opening angle will be:

$$\theta_{MHD} \simeq \frac{c_s}{c} \simeq 3 \times 10^{-3} \text{ rad} \simeq 0.2^\circ, \quad (54)$$

where c_s is the sound speed in the helium core, $\simeq 10^8 \text{ cm s}^{-1}$. This corresponds to a solid angle of $\Delta\Omega/4\pi \simeq 2 \times 10^{-6}$. The UMHDW may then propagate up the axis in a radiation-dominated jet with smaller cross section than the original matter-dominated jet. If this is the case, the channel carved by the UMHDW may remain substantially smaller than the characteristic wavelength of the UMHDW until the waves break out of the surface of the star.

To avoid the baryon loading problem and to generate a γ -ray burst with large Lorentz factor, the hole through which the bulk of the UMHDW emerge should contain less than

$$\Delta M \lesssim \frac{10^{52} \text{ ergs}}{\Gamma^2 c^2}, \quad (55)$$

(Rees & Mészáros 1992). For $\Gamma \gtrsim 100$, the requirement is $\Delta M \lesssim 10^{27} \text{ g} \simeq 5 \times 10^{-7} \text{ M}_\odot$. It is difficult to estimate whether the UMHDW jet will entrain such a small amount of matter. There are two qualitative possibilites for the propagation of the UMHDW jet. There may be a density minimum along the axis of the previous matter jet as suggested by the calculations of Khokhlov et al (1999). If the UMHDW propagate through such a low density axial channel, they will naturally emerge somewhat more collimated than the original jet. On the other hand, if the jet is denser than the surrounding stellar matter and the jet acts like a plug on the axis, the UMHDW might propagate in a cylindrical blanket around the plug. They would then emerge with an annular cross section. This would result in yet another complication for predicting the observational aspects of such an event.

If the matter jet leaves behind a region of very low baryon density, or the UHMDW emerge beyond the end of the more slowly propagating matter jet, then the propagating UMHDW could enter a low density region in which the density of the environment is lower then the critical Goldreich-Julian density (Goldreich & Julian 1969; Shapiro & Teukolsky 1983) of

$$\rho_{GJ} \simeq 10^{-6} \text{ g cm}^{-3} \left(\frac{B(R)}{10^{16} \text{ G}} \right) \left(\frac{P}{1 \text{ ms}} \right)^{-1}, \quad (56)$$

where the flux-freezing and force free conditions are broken (see equation 30). At this point, the transition from UMHDW to LAEMW can be made. Reconnection of magnetic fields and rapid acceleration of particles in a pair plasma follows (e.g. Asseo, Kennel & Pellat 1978; Usov 1992; Michel 1984; Thompson 1994; Blackman, Yi & Field 1995). The pair plasma could in principle be accelerated to a high bulk Lorentz factor up to $\simeq 10^5$, although the exact value of the maximum bulk Lorentz factor could be significantly lower than this due to baryon loading and other complicated factors reducing acceleration efficiency (cf. Usov 1992, 1994).

The rest-frame γ -ray luminosity would be determined by the electromagnetic power, i.e.

$$L_{obs} \simeq f_\gamma L_{MHD} \simeq 4 \times 10^{51} \text{ erg s}^{-1} \left(\frac{f_\gamma}{0.1} \right) \left(\frac{\Omega_{NS}}{10^4 \text{ s}^{-1}} \right)^4 \left(\frac{R_{NS}}{10 \text{ km}} \right)^6 \left(\frac{B_{NS}}{10^{16} \text{ G}} \right)^2, \quad (57)$$

where f_γ is the γ -ray emission efficiency factor. During this phase, the pulsar rotation energy is used to power the γ -ray burst while the spin of the pulsar slows down as

$$\begin{aligned} \Omega_{NS} &= 10^4 \text{ s}^{-1} \left(\frac{\Omega_{NS,i}}{10^4 \text{ s}^{-1}} \right) \\ &\times \left[1 + 6 \times 10^{-2} \left(\frac{M_{NS}}{1.5M_\odot} \right)^{-1} \left(\frac{\Omega_{NS,i}}{10^4 \text{ s}^{-1}} \right)^2 \left(\frac{R_{NS}}{10 \text{ km}} \right)^4 \left(\frac{B_{NS}}{10^{16} \text{ G}} \right)^2 \left(\frac{t - t_{em,i}}{1 \text{ s}} \right) \right]^{-1/2}, \end{aligned} \quad (58)$$

where $\Omega_{NS,i}$ is the initial spin frequency of the neutron star and $t_{em,i}$ is the initial time of the spin-down phase driven by the electromagnetic dipole radiation emission. The time evolution of the γ -ray luminosity is then given by

$$L_{obs} \simeq 4 \times 10^{51} \text{ erg s}^{-1} \left(\frac{f_\gamma}{0.1} \right) \left(\frac{\Omega_{NS,i}}{10^4 \text{ s}^{-1}} \right)^4 \left(\frac{R_{NS}}{10 \text{ km}} \right)^6 \left(\frac{B_{NS}}{10^{16} \text{ G}} \right)^2 \times \left[1 + 6 \times 10^{-2} \left(\frac{M_{NS}}{1.5M_\odot} \right)^{-1} \left(\frac{\Omega_{NS,i}}{10^4 \text{ s}^{-1}} \right)^2 \left(\frac{R_{NS}}{10 \text{ km}} \right)^4 \left(\frac{B_{NS}}{10^{16} \text{ G}} \right)^2 \left(\frac{t - t_{em,i}}{1 \text{ s}} \right) \right]^{-2}. \quad (59)$$

The LAEMW-powered γ -ray emission stage shows a characteristic initial phase during which the luminosity remains nearly constant. This phase is expected to be followed by a phase in which the luminosity decreases as $\propto t^{-2}$. As shown in Blackman and Yi (1998), the γ -ray emission via the synchrotron-Compton process gives the luminosity of the peak $\propto t^{-1}$. We note that if the proton fraction changes after some complicated acceleration processes, then the synchrotron-Compton emission process and the above scalings could be significantly different.

To get γ -rays directly from the LAEMW, there must be a density in the environment that is below the Goldreich-Julian density so the currents cannot be supported in the plasma and a pair cloud is spontaneously generated. The Goldreich-Julian density scales with the ambient magnetic field. If that field falls off like R^{-1} in the propagating Poynting flux, then beyond the helium core at $R \gtrsim 10^6 \text{ km}$, the Goldreich-Julian density will be (equation 56) $\rho \lesssim 10^{-11} \text{ g cm}^{-3}$. Densities this low might occur immediately beyond the star (and the jet) if the star is embedded only in the interstellar medium, but it is likely that the progenitor of a Type Ib/c supernova is surrounded by a wind or other mass resulting from mass-loss processes. For a constant velocity wind at $10^8 v_8 \text{ cm s}^{-1}$ carrying mass at a rate $10^{-5} \dot{M}_{-5} M_\odot \text{ yr}^{-1}$, the density is

$$\rho = 5 \times 10^{-9} \text{ g cm}^{-3} \dot{M}_{-5} v_8^{-1} R_5^{-2}, \quad (60)$$

with radius in units of 10^5 km . This density will be less than the Goldreich-Julian density for $R \gtrsim 10^6 \text{ km}$, several times the radius of the helium core.

Although the possibility of γ -ray emission by pair cascade is not ruled out, it is important to consider how the UMHDW/LAEMW escape from the stellar core and associated processes. A sub-relativistic matter-dominated MHD jet leaves behind a baryon-rich environment. The common statement is that if the baryon loading is too high a γ -ray burst of observed properties cannot be produced. We show below that this is not necessarily the case.

Since the plasma density (baryons, leptons, and photons) is high, the perfect MHD condition is always maintained throughout the phase when the UMHDW propagate within the stellar core. Due to the high radiation density and the large amount of high temperature stellar material, the optical depth for outgoing radiation is exceedingly high. This makes the γ -ray emission efficiency very low. Under these circumstances, the bulk of the energy supplied by the UMHDW may be used to accelerate shocks parallel to the rotation axis and the axis of the matter jet. After the shock breaks out, γ -ray emission could occur as the kinetic energy of the shock is dissipated into

particle acceleration and subsequent synchrotron-Compton emission. We note one important difference between this situation and that of the original Colgate mechanism. As noted in §7.1, in the Colgate mechanism there is an impulsive input of energy in the stellar core and a single shock that propagates outward. This leads to a single, short pulse of hard emission. This process may, indeed, work when the shock driven by the UMHDW first encounters the steep density gradient at the stellar surface or at the tip of the jet, but in the current situation the pulsar and the UMHDW/LAEMW continue to drive shocks for an extended time. The density profile will be altered as the continuing shock energy is deposited and that reaction must be considered self-consistently.

It is difficult to estimate the γ -ray emission efficiency in this process. Even for a very low efficiency $\lesssim 10^{-4}$, however, a γ -ray burst event such as SN 1998bw/GRB980425 could be generated provided that the UMHDW power is large enough, $\gtrsim 10^{51}$ erg s $^{-1}$ as has been assumed. If the efficiency of production of γ -rays approaches unity, then, with appropriate collimation, a γ -ray burst detectable at cosmological distances could be produced. Aside from the complex details of the γ -ray production, the overall bolometric luminosity evolution is expected to follow the time evolution described by equation (59).

In this context, we take note of circumstances when the deposition of energy into baryons does not necessarily doom the production of a γ -ray burst. As noted by Protheroe & Bednarek (1999), if protons can be accelerated to energies where pion production is efficient, then γ -rays can be produced in the subsequent pion decay. To reach this regime, the protons must be given at least mildly relativistic energies, in excess of 1 GeV. This requires that the energy of the UMHDW/LAEMW be shared with less than $0.005 M_{\odot} E_{52}$ of baryons. The shock driven by the UMHDW/LAEMW should boost some protons to even higher bulk velocities and the energy deposited by the UMHDW/LAEMW will plausibly go into a mass less than the total mass of the precursor subrelativistic jet if their effects are concentrated in the relatively low density matter along the axis of the jet or the matter immediately surrounding the jet. Another alternative is that the protons will be accelerated by the momentum associated with the Poynting flux. Deposition of the energy of the pulsar spindown via UMHDW selectively into the bulk motion of protons with energies substantially above the pion production threshold is thus a distinct possibility. Thus rather than being a handicap, the “baryon loading” of the jet could be an advantage in the conversion of pulsar energy into γ -ray energy.

Although a quantitative calculation is required, we envisage a process by which the protons are accelerated to high energy with the associated efficient production of pions. To produce the pions, the protons must collide with a “target.” In the present context, a good candidate for the target is the wind expected to be present around the progenitor. The stopping length for proton-proton interaction to produce pions is about $100 \text{ cm}^2 \text{ gm}^{-1}$. For the density given by equation (60), the column depth of the wind is

$$l = 50 \text{ g cm}^{-2} \dot{M}_{-5} v_8^{-1} R_5^{-1}, \quad (61)$$

with radius again in units of 10^5 km. Thus a substantial wind might provide the stopping medium for the conversion of high-velocity protons to pions in the vicinity of the surface of a hydrogen-stripped star.

The pions produced when the high-energy protons collide with the wind then decay and produce very energetic γ -rays. Note that the stopping length for γ -rays is about $30 \text{ cm}^2 \text{ gm}^{-1}$, so a “target” that stops protons will be somewhat optically thick to γ -rays. These pion-decay high energy γ -rays do not immediately correspond to the observed γ -rays in a γ -ray burst. Rather, they induce a pair cascade through photon-photon collision. This pair fireball could then produce the observed γ -ray burst. The strength of a γ -ray burst produced in this way will depend on the relative efficiency of production of pions versus neutrinos. The latter would be an energy sink for this process.

Note that equation (61) neglects any affect of the ambient or stellar magnetic fields or those associated with the UMHDW/LAEMW. If the field were strong and the interparticle spacing small, the shock generated by the UMHDW/LAEMW could be collisionless and there might be no pion production. Alternatively, if the shock accelerates protons into a small magnetic field then this estimate would be valid. As an example, if the field scales as R^{-2} from a central dipole of 10^{16} G, then the Larmor radius near the surface of the star would be about 10^{-4} cm. If the field is associated with the LAEMW and scales as R^{-1} , then the Larmor radius would be about 10^{-8} cm. At the density given in the wind by equation (60), the interparticle spacing is about 10^{-5} cm. Thus gyrating protons might still collide in the 10^8 G dipole field with an effective path length enhanced by the gyration, and equation (61) could be a lower limit, or they could be rendered collisionless in the 10^{12} G field associated with the LAEMW. On the other hand, since the UMHDW/LAEMW could dominate the dynamics of the protons, the idea of a shock among particles carried along by the LAEMW might be moot anyway (Usov 1999). Clearly, the physics associated with these processes is complex and in need of deeper understanding. The only point to be made here is that the physics of proton acceleration and pion production in the wind might be relevant and needs further study.

8. Discussion and Conclusions

We have shown that the contraction phase of a proto-neutron star could result in a substantial change in the physical properties of the environment. The following parameters are relevant:

- $P_{PNS} \simeq 25 \text{ ms}$
- $R_{PNS} \simeq 50 \text{ km}$
- $R_{A,PNS} \approx 150 \text{ km}$
- $R_{LC} \simeq 10^3 \text{ km}$
- $I_{PNS} = kR_{PNS}^2 M_{NS} \simeq 3 \times 10^{46} \text{ c.g.s.}$
- $\tau_{\text{conv}} \simeq 1 \text{ ms}$
- $P_{NS} \simeq 1 \text{ ms}$
- $R_{NS} \simeq 10 \text{ km}$
- $R_{A,NS} \approx 150 \text{ km}$
- $R_{LC} \simeq 50 \text{ km}$
- $I_{NS} = kR_{NS}^2 M_{NS} \simeq 10^{45} \text{ c.g.s.}$
- $\tau_{\text{conv}} \simeq 1 \text{ ms}$

- $R_0 \sim P_{PNS}/\tau_{\text{conv}} \gg 1$
→ no dynamo
- magnetic field amplified within ~ 10 s by linear amplification or flux-freezing
- MHD jet & torque action
- hole punched & envelope torqued
- $R_0 \gtrsim 1$
→ $\alpha - \Omega$ dynamo
- exponential growth of field
- UMHDW generated: $\vec{S} = \frac{c}{4\pi}(\vec{E} \times \vec{B})$
- relativistic jet
envelope accelerated and heated by waves

When the rotating magnetized neutron star first forms there is likely to be linear amplification of the magnetic field and the creation of a matter-dominated jet, perhaps catalyzed by subrelativistic MHD effects, up the rotation axis. The energy of the proto-neutron star is sufficient to power a significant matter jet, but unlikely to generate a strong γ -ray burst. The matter jet could generate a smaller γ -ray burst as seems to be associated with SN 1998bw and GRB 980425 by the Colgate mechanism as it emerges and drives a shock down the stellar density gradient in the absence of a hydrogen envelope, e.g., in a Type Ib/c supernova. As the neutron star cools, contracts, and speeds up, two significant things happen. One is that the rotational energy increases. The energy becomes significantly larger than required to produce a supernova and sufficient, in principle, to drive a cosmic γ -ray burst if the collimation is tight enough and losses are small enough. In addition, the light cylinder contracts significantly, so that a stationary dipole field cannot form and the emission of strong UMHDW occur. Tight collimation of the original matter jet and of the subsequent flow of UMHDW in a radiation-dominated jet is expected.

The UMHDW will propagate as intense low frequency, long wavelength Poynting flux. They may be isotropized to act as a relativistic fluid, but not thermalized since they have a frequency much less than the plasma frequency. The UMHDW “bubble” could be strongly Rayleigh-Taylor unstable, but still may propagate selectively with small opening angle up the rotation axis as an UMHDW jet. Alternatively, the impulsive production of UMHDW could render the stellar matter nearly irrelevant as a confining medium. If a UMHDW jet forms, it can drive shocks which may selectively propagate down the axis of the initial matter jet or around the perimeter of the matter jet. The shocks associated with the UMHDW jet could generate γ -rays by the Colgate mechanism as they propagate down the density gradient at the tip of the jet or there could be bulk acceleration of protons to above the pion production threshold. The protons could produce copious pions upon collision with the surrounding wind, thus triggering a cascade of high energy γ -rays, pairs, and lower-energy γ -rays in an observable γ -ray burst. Yet another alternative is that the UMHDW could eventually propagate into such a low density environment that they directly induce LAEMW and pair cascade (see also Thompson & Madau 1999).

The radiation-dominated jet cannot form for several seconds as the neutron star contracts, spins up, and generates a large magnetic field, but then it propagates faster than the matter-dominated jet. In this circumstance, the matter jet could precede or follow the radiation-dominated jet. In the former case an X-ray precursor could be generated. In the latter case the matter jet might not be conspicuous at all.

There are a number of reasons why the processes we have outlined may not be as effective as we have assumed. One is that the rotation of the neutron star may prevent contraction to the high densities and rotation rates on the timescales we have assumed (Fryer & Heger 1999). This could affect the convection in the neutron star and hence the generation of the magnetic field by the $\alpha - \Omega$ dynamo mechanism. This is a complex issue, of course, since the presence of the magnetic field will lead to energy loss and conditions of greater contraction, as we have invoked here (Symbalisty 1984).

Another complication we have ignored is that the mean dipole field that forms may have its axis tilted with respect to the spin axis. This may not be a critical factor, since the subsequent dynamics may be dominated by the density distribution surrounding the neutron star that is predominantly set by the angular momentum, not the magnetic field. The question of what fraction of the pulsar energy goes to drive quasi-spherical expansion and what fraction propagates as co-linear UMHDW clearly requires greater study. We also noted that if the UMHDW are thermalized in the stellar core the result will be the copious production of electron/positron pairs. At first, such a pair cloud will behave like a relativistic fluid so there may be little difference in the dynamics. As Rayleigh-Taylor instabilities ensue, the pairs might get mixed with ordinary matter and the positrons annihilate. Even if the pair bubble escapes up the axis, as we envisage for non-thermalized UMHDW, it will expand as it propagates, especially after breaking through the stellar surface. Much of the thermal energy could then be lost to kinetic energy by adiabatic expansion. If there is no “working surface” with which this pair cloud could collide, then much of the energy could be lost. On the other hand, the sort of wind expected (equation 60) would give a stopping length (equation 61) that will easily stop the pairs. The issue would then be the efficiency of conversion of their kinetic and rest mass energy into γ -rays.

There is an interesting question of the opposite sort concerning the possibility that the production of large energy in UMHDW could be too effective. For instance, if an UMHDW jet could propagate through the naked core of a Type Ic, but would get stopped and dissipated in a red supergiant, then we might find that the explosion energy of Type II is systematically higher than that of Type Ic. The fact that there is no clear evidence for this may suggest that the production of 10^{52} ergs in UMHDW is not a common occurrence in core collapse, but this question requires further study.

The question of whether or not pulsar spin-down will produce a γ -ray burst depends on such factors as the initial rotation rate, the strength of the dipole field that evolves, the tilt of such a field compared to the spin axis, and the density of the progenitor wind. Issues of uncertain

physics aside, it is clear that this mechanism might not be robust in the production of γ -ray bursts, but might produce γ -ray bursts of varying strength depending on natural variation in the circumstances of a given collapse event.

Any γ -rays emitted by any of these processs are likely to be strongly collimated. The luminosity of the emitted radiation will depend on the geometry of that emission. We have noted here that the energy produced by the spin-down of the pulsar could emerge from the stellar surface along the axis of a low-density matter jet, or in an annulus surrounding a high density jet. Either of these cases will give a Lorentz factor that depends strongly on the aspect angle of the observer. Computation of the resulting luminosity is thus distinctly non-trivial.

Simple γ -ray burst models invoking collimation assume that the energy in a collimated burst scales simply as $\Delta\Omega/4\pi$ compared to that deduced in an isotropic geometry. This assumes that the collimated jet nevertheless expands as the section of a sphere with a cross section scaling as R^2 . We note that if the relativistic flow is truly collimated, this may yet be an overestimate of the energy required to power a given observed γ -ray burst. The calculation of a jet emerging from a stellar core by Khokhlov et al. (1999) shows that the dynamical jet that precedes any possible γ -ray burst is nearly linearly collimated and does not expand in cross section as R^2 . In the absence of an understanding of the dynamics of the subsequent flow of UMHDW/LAEMW, we do not know the geometry of the relativistic flow, but if the cross sectional area grows less strongly than R^2 , then the solid angle $\Delta\Omega/4\pi$ will be a *decreasing* function of distance. The energetics (and hence the derived rates of occurrence) of γ -ray bursts will depend not only on the fact, but the geometry of the collimation. If the cross section expands less rapidly than R^2 , then substantially less energy may be required to produce a given observed γ -ray burst.

The observed nature of any γ -ray burst will depend on whether or not one is directly witnessing a strongly relativistic bulk flow. There is general agreement that the observed afterglows of the cosmic γ -ray bursts represent relativistic blast waves. The question of whether that is true or not for the primary γ -ray burst is still in contention with models based on internal shocks in a “central machine,” in which the burst duration is the fundamental physical time scale of the energy production process (e.g. Piran 1999, and references therein), vying against models invoking relativistic flows and external shocks for which there is strong Lorentz contraction of the timescales between the production mechanism and the observer (e.g. Dermer, Böttcher & Chiang 1999). Fenimore & Ruiz (1999) have recently argued that a central machine is favored (see also Heinz & Begelman 1999).

One of the implications of this uncertainty is the location of the γ -ray burst. If an external blast wave is involved, then a γ -ray burst with a time scale in the observer frame of t_{obs} has a propagation distance of $\Gamma^2 ct_{obs}$. For a γ -ray burst of 10 s this distance is about 3×10^{15} cm for a Lorenz factor of $\Gamma \simeq 100$. This is much larger than the radius of the hydrogen-deficient stellar core we are considering here, $R_{core} \simeq 10^{10}$ cm. It is not clear why the energy produced by shocks and UMHDW/LAEMW would be dumped at a radius as large as 10^{15} cm. Shocks should deposit

their energy as they emerge from the star, and the density falls below the Goldreich-Julian density at only 10^{11} cm even for a relatively dense wind. If relativistic protons are generated by shocks or other mechanisms, they could also plausibly be stopped near the stellar surface by a substantial wind. Thus for all the mechanisms we sketch here, the most logical site of the production of any γ -ray burst is near the stellar surface. This implies that if this general process of pulsar spindown from massive star core collapse has any role in producing γ -ray bursts it will most plausibly serve as a “central engine.” We note that in this case, the natural time scale for any γ -ray burst is about 5 to 10 sec, the cooling, spin down time for the neutron star. On the other hand, the eruption of shocks and UMHDW/LAEMW from the stellar surface may occur in a region small compared to the stellar surface, so considerably shorter timescales might be manifest for central burst peaks. If the emission from the pulsar is prolonged, then one might also witness time scales associated with the processes of production of the UMHDW, for instance various instabilities, as well. The shortest time scales of any substructure would be $\simeq 10^{11}$ cm/ $\Gamma^2 c$ or about 0.3 ms for $\Gamma = 100$.

The deposition of a large amount of energy at the stellar surface could, of course, result in a subsequent relativistic blast wave and associated afterglow. The processes we are discussing would produce a maximum “isotropic equivalent” energy of $4\pi E/\Delta\Omega \simeq 10^{54}$ erg for $E_{52} \simeq 1$ and $\Delta\Omega/4\pi \simeq 0.01$. The external mass required to decelerate this energy, $E\Gamma^{-2}c^{-2}$, requires a spherically-distributed mass of $\simeq 5 \times 10^{-5} E_{52} M_\odot$ for $\Gamma = 100$. The mass in the wind out to a radius R is

$$M_{wind} = 3 \times 10^{-6} M_\odot \dot{M}_{-5} v_8^{-1} R_{15}, \quad (62)$$

with R in units of 10^{15} cm. Thus energy emitted by the processes we have outlined will not be decelerated until a radius of about 10^{16} cm. Even ignoring issues of entrainment and asymmetries in the wind density profile, this radius is sensitive to the energy of the burst, the degree of collimation, the value of the Lorentz factor and the parameters of the wind.

The point of this paper is not to establish that core collapse and pulsar formation will lead to γ -ray bursts, but to establish that this environment gives a framework in which to quantitatively address questions of physics that are germane to the nature of the core collapse process and to potential γ -ray production. It seems very clear that rotation and magnetic fields have a strong potential to create axial matter-dominated jets that will drive strongly asymmetric explosions for which there is already ample observational evidence in Type II and Type Ib/c supernovae, their remnants, and in the pulsar velocity distribution. The potential to also create strong flows of UMHDW/LAEMW serves to reinforce the possibility to generate asymmetric explosions. These asymmetries will affect nucleosynthesis and issues such as fall-back that determine the final outcome to leave behind neutron stars or black holes. In addition, the presence of matter-dominated and radiation-dominated jets might lead to bursts of γ -rays of various strengths. The issue of the nature of the birth of a “magnetar” in a supernova explosion is of great interest independent of any connection to γ -ray bursts. Highly magnetized neutron stars might represent one out of ten pulsar births. Production of a strong γ -ray burst might be even more rare.

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